Topological Models for (Full) Belief and Belief Revision

Aybüke Özgün

LORIA, CNRS-Université de Lorraine

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Joint work with Alexandru Baltag, Nick Bezhanishvili and Sonja Smets.

Background Notions

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An open set containing $x \in X$ is called *open neighborhood* of x.

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$$\begin{aligned} \mathrm{Int} : \mathcal{P}(X) &\rightarrow \mathcal{P}(X) \\ \mathrm{Int}(A) &\mapsto \text{the largest open set contained in } A \end{aligned}$$

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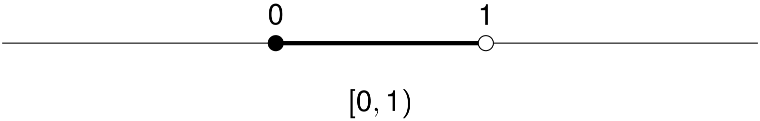
- $\mathrm{Cl}(A) = X \setminus \mathrm{Int}(X \setminus A)$

Example:

Consider the real line and the topology of open intervals and their countable unions on $\mathbb{R}$.
Let $A = [0, 1)$



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We can think of

- open sets $U \in \tau$ as pieces of evidence, and
- open neighborhoods of the actual world as pieces of **truthful** evidence.

Moreover, ...

Theorem (McKinsey and Tarski, 1944)

S4 *is sound and complete wrt the class of all topological spaces.*

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but **not** necessarily *negatively introspective*

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Downsides:

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2. it can easily be *"Gettierized"*:

$$K\varphi := B\varphi \wedge \varphi$$

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2. how to extend this setting to static and dynamic belief revision?

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Epistemic-Doxastic Axioms	
$K(\varphi \to \psi) \to (K\varphi \to K\psi)$	Knowledge is additive
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$K\varphi \to KK\varphi$	Positive introspection for K
$B\varphi \to \neg B\neg\varphi$	Consistency of belief
$B\varphi \to KB\varphi$	(Strong) positive introspection of B
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$K\varphi \to B\varphi$	Knowledge implies Belief
$B\varphi \to BK\varphi$	Full Belief

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From φ and $\varphi \to \psi$ infer ψ.	Modus Ponens
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- Belief as *subjective certainty*: *an agent "fully" believes* φ *iff she believes that she knows it.*

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3. **S4.2** as the logic of knowledge

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A space (X, τ) is called extremally disconnected if the closure of each open subset of X is open.

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Given an extremally disconnected space (X, τ), we interpret belief as *the closure of the interior operator*:

$$[\![B\varphi]\!] = \mathrm{Cl}(\mathrm{Int}([\![\varphi]\!]))$$

The Most General Extensional Semantics for **KB**

- An extensional semantics for $\mathcal{L}_{KB}$: $(X, \mathrm{K}, \mathrm{B})$
  - $\mathrm{K} : \mathcal{P}(X) \to \mathcal{P}(X)$
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- A topological extensional semantics for $\mathcal{L}_{KB}$:

$$(X, \mathrm{K}^{\tau}, \mathrm{B}^{\tau}) := (X, \mathrm{Int}, \mathrm{Cl}(\mathrm{Int}))$$

The Most General Extensional Semantics for **KB**

Theorem (Topological Representation Theorem)

An extensional semantics $(X, \mathrm{K}, \mathrm{B})$ *validates all the axioms and rules of Stalnaker's system* **KB** *iff it is a topological extensional semantics given by an extremally disconnected topology* τ *on* X.

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- Known: **S4.2** is sound and complete wrt the class of extremally disconnected spaces.

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Theorem

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Conditional Beliefs

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- $B^{\psi}\varphi :=$ *If the agent would learn ψ she would come to believe that φ was the case before the learning.*

- For any topological model $\mathcal{M} = (X, \tau, \nu)$ based on an extremally disconnected space (X, τ),

$$[\![B\varphi]\!]^{\mathcal{M}} = \mathrm{Cl}(\mathrm{Int}([\![\varphi]\!]^{\mathcal{M}})) = \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}([\![\varphi]\!]^{\mathcal{M}}))).$$

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- The Basic Topological Semantics for CB:

$$[\![B^{\varphi}\psi]\!] = \mathrm{Cl}([\![\varphi]\!]^{\mathcal{M}} \cap \mathrm{Int}([\![\varphi]\!]^{\mathcal{M}} \to [\![\psi]\!]^{\mathcal{M}})).$$

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- The Refined Topological Semantics for CB:

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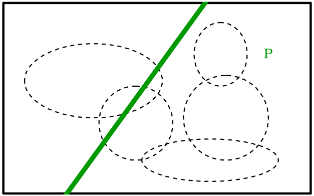

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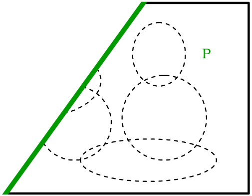

(P, τ_P)

Dynamic Belief Revision: Updates

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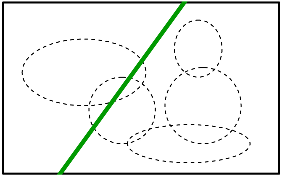
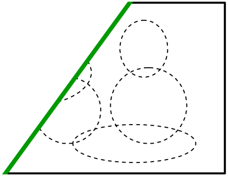


$\xrightarrow{\langle !\varphi \rangle}$



- $[\![\varphi]\!] = [\![\varphi]\!]^{\mathcal{M}}$
- $\tau_{[\![\varphi]\!]} = \{U \cap [\![\varphi]\!] : U \in \tau\}$
- $\nu_{[\![\varphi]\!]}(p) = \nu(p) \cap [\![\varphi]\!]$ for any $p \in \mathrm{Prop}$

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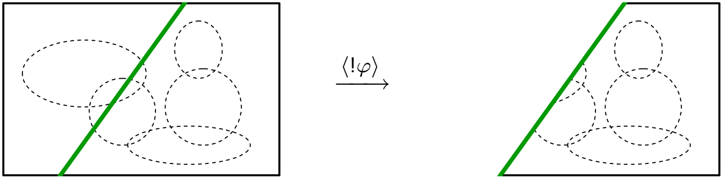
- $\langle !\varphi \rangle \psi := \varphi$ *is true and after the agent learns* φ, ψ *becomes true.*

$\mathcal{M} = (X, \tau, \nu)$ $\mathcal{M}_{\varphi} = ([\![\varphi]\!], \tau_{[\![\varphi]\!]}, \nu_{[\![\varphi]\!]})$



- Given a topological model $\mathcal{M} = (X, \tau, \nu)$, the additional semantic clause reads

$$x \in [\![\langle !\varphi \rangle \psi]\!]^{\mathcal{M}} \text{ iff } x \in [\![\varphi]\!] \text{ and } x \in [\![\psi]\!]^{\mathcal{M}_{\varphi}}.$$

Future Work

- Multi-agent case
- Action models
- Dynamic attitudes
- Relation with topo-logic and *effort modality*

Thank you!