LEARNING BY ERASING IN DYNAMIC EPISTEMIC LOGIC

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Language and Automata Theory and Applications
April 3rd 2009

OUTLINE

Outline

The Two

Operational versus epistemological view

Formal attempts to grasp the phenomenon of epistemic change:

- formal learning theory (LT) with scientific discovery,
- belief-revision theory and dynamic epistemic logic (DEL).

LEARNING THEORY

- Identification in the limit [Gold 1967].
- Grammar inference, applications in syntax.
- Acquisition of semantics of natural language.
- Modeling the process of scientific inquiry.
- Model-theoretic learning with the belief-revision link.

Modal approach to epistemic change

- Epistemology!
- Language to discuss epistemic states of agents.
- Formalizing dynamics of knowledge (AGM).
- Modeling in dynamic epistemic logic.

Why merge?

- LT motivations.
  - BR-based ‘simple-minded’ learning.
  - (Epistemic) modal logic for learning.
- DEL motivations.
  - Operational knowledge.
  - The role of uncertainty.

OUTLINE

Learning Theory
Identification

1. A class of possible worlds.
2. Nature chooses one of them.
3. Nature generates data about the world.
4. From inductively given data Scientist draws conjectures.
5. A new information comes in, Scientist gives a hypothesis.
6. Scientist gets to a correct hypothesis.

Success Condition as a Parameter

1. Identification in the limit.
2. Finite identification.
3. Learning by erasing.

Identification in the limit

Definition

We say that a learning function L:

1. identifies $S \in C$ in the limit on ε iff there is a number k, such that for co-finitely many m, $L(\varepsilon|m) = k$ and $k \in I_S$;
2. identifies $S \in C$ in the limit iff it identifies S in the limit on every ε for S;
3. identifies C in the limit iff it identifies in the limit every $S \in C$.

Finite identification

Definition

We say that a learning function L:

1. finitely identifies $S \in C$ on ε iff, when inductively given ε, at some point L outputs a single k, such that $k \in I_S$, and stops;
2. finitely identifies $S \in C$ iff it finitely identifies S on every ε for S;
3. finitely identifies C iff it finitely identifies every $S \in C$.

Learning by Erasing

What if:

- we interpret the outputs negatively,
- introduce an ordering on the hypothesis space,
- and take the actual conjecture to be the minimal element?

References:

Freivalds, R., Zeugmann, T.: Co-learning of recursive languages from positive data. 1996

Lange, S., Wiehagen, R., Zeugmann, T.: Learning by erasing. 1996

Freivalds, R., Karpinski, M., Smith, C., Wiehagen, R.: Learning by the process of elimination. 2002

Learning by Erasing

Definition (Function Stabilization)

In learning by erasing we say that a function stabilizes to number k on environment ε if and only if for co-finitely many $n \in \mathbb{N}$:

$$k = min\{\mathbb{N} - \{L(\varepsilon|0), \ldots, L(\varepsilon|n)\}\}.$$

Definition

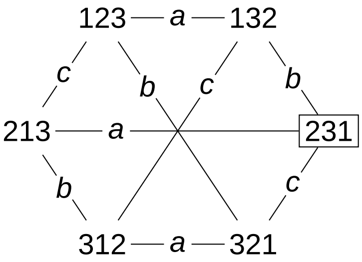
We say that a learning function L:

1. learns $S \in C$ by erasing on ε iff L stabilizes to k on ε and $k \in I_S$;
2. learns $S \in C$ by erasing iff it learns by erasing S from every ε for S;
3. learns C by erasing iff it learns by erasing every $S \in C$.

OUTLINE

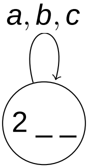
EXAMPLE

Agents $A = \{a \text{ (Anne)}, b \text{ (Bob)}, c \text{ (Carl)}\}$
Cards: 1, 2, 3.
E.g. 231 means that Anne has 2, Bob has 3 and Carl has 1.



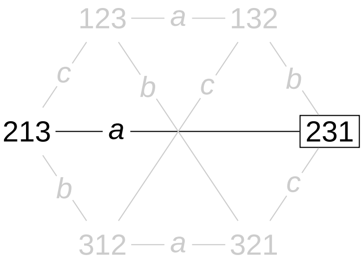
EXAMPLE

Anne shows her card to all the players publicly.



EXAMPLE

After Anne's announcement.



Dynamic Epistemic Logic

Definition (Epistemic Model)

Epistemic model M is a triple $\langle W, \{\sim_i\}_{i \in A}, V \rangle$, where W is a set of possible worlds, for each $i \in A$, $\sim_i \subseteq W \times W$ is an indistinguishability relation and $V : Atom \rightarrow \wp(W)$ is a valuation.

Definition

An event model E is a triple $\langle S, \{\rightarrow_i\}_{i \in A}, pre \rangle$, where S is a set of worlds, for each $i \in A$, $\rightarrow_i \subseteq S \times S$, and $pre : S \rightarrow Atom$ is a pre-condition function which indicates what *pre-condition* a world has to satisfy to enable the event to take place.

Epistemic Update

Definition (Product Update)

Let $M = \langle W, \{\sim_i\}_{i \in A}, V \rangle$ and $E = \langle S, \{\rightarrow_i\}_{i \in A}, pre \rangle$. The product update $M \otimes E$ is the epistemic model $M' = \langle W', \{\sim'_i\}_{i \in A}, V' \rangle$ such that:

- $W' = \{(w, s) | w \in W, s \in S \text{ and } M, w \models pre(s)\}$,
- $(w, s) \sim_i (w', s')$ iff $w \sim_i w'$ and $s \rightarrow_i s'$,
- $V'((w, s)) = V(w)$.

Definition (Public Announcement)

The public announcement of a proposition p is the event model $E_p = \langle S, \{\rightarrow_i\}_{i \in A}, pre \rangle$, such that $S = \{e\}$ and for each $i \in A$, $e \rightarrow_i e$ and $pre(e) = p$.

Dynamic Doxastic Logic

Definition (Epistemic Plausibility Model)

Let *Atom* be a set of atomic propositions and A — a set of agents. Epistemic plausibility model E is a quadruple: $\langle W, \{\sim_i\}_{i \in A}, \{\leq_i\}_{i \in A}, V \rangle$, where W is a set of possible worlds, for each $i \in A$, $\sim_i \subseteq W \times W$ is an indistinguishability relation, $\leq_i \subseteq W \times W$ is a preference relation and $V : Atom \rightarrow \wp(W)$ is a valuation.

LET'S MERGE

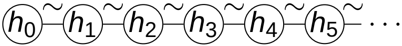
Outline

LEARNING IN DEL AND DDL

- One-step *learning that* φ, followed by epistemic update.
- In LT the incoming information $\neq$ thing being learned.
- Two-sorted models.
- Hypothesis as the set of sequences of events.
- Events are announcements of elements of sets,
- and not hypotheses themselves.

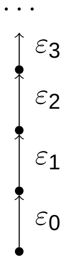
Finite identification in DEL

Initial Epistemic Model

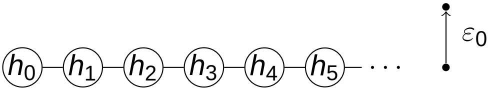


Finite identification in DEL

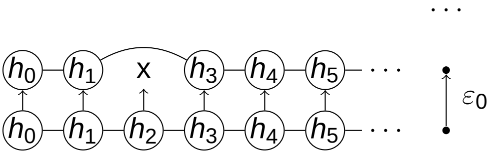
Environment ε consistent with h_3



Finite identification in DEL
Confrontation with data



Finite identification in DEL
Epistemic update



Finite identification in DEL

Theorem

Finite identifiability can be modeled in DEL.

We use:

- epistemic states for hypotheses;
- infinite sequences of announcements for environments;
- epistemic update for the progress in eliminating uncertainty over hypothesis space.

Scientist succeeds in finite identification of S from ε iff:

1. there is a finite initial segment of ε, $\varepsilon|n$, such that
2. the domain of the $\varepsilon|n$-generated model contains only h_k
3. and $k \in I_S$.

There is a finite step of the iterated epistemic update along ε, that eliminates uncertainty.

Learning by Erasing in DDL

Theorem

Learning by erasing can be modeled in DDL.

We use:

- epistemic states for hypotheses;
- infinite sequences of announcements for environments;
- epistemic update for the elimination of hypotheses;
- preference relation for the underlying ordering of hypotheses;
- at each step, the most preferred hypothesis is conjectured.

Scientist learns S by erasing from ε iff

1. there is n such that for every $m > n$,
2. the most preferred state of $\varepsilon|m$-generated model is h_k,
3. and $k \in I_S$.

Outline

Conclusions and Further Work

Some types of inductive inference modeled in DEL and DDL.

- establish a tighter correspondence
- check LT for different kinds of events
- semantics $\rightarrow$ language and axioms for inductive inference
- non-introspective operational knowledge and uncertainty
- LT in Epistemic and Doxastic **Temporal** Logic:
  1. an ETL model H satisfies FIN iff $A(i \implies \forall FKi)$
  2. a DTL model H satisfies ERASE iff $\exists \leq A(i \implies \forall FGBi)$

Dégremont, C., Gierasimczuk, N.: Can doxastic agents learn? On the temporal structure of learning. 2009

THANK YOU